Discrete Mathematics (2)

Let \( S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \), and the binary relation \( R = \{(a, b) \mid |a - b| \leq 1 \text{ or } 2 \text{, and } a, b \in S\} \).

For instance, \( 2 \mathrel{R} 4 \), \( 7 \mathrel{R} 6 \), but not \( 5 \mathrel{R} 2 \).

1. Determine whether \( R \) is reflexive.
2. Determine whether \( R \) is symmetric.
3. Determine whether \( R \) is anti-symmetric.
4. Determine whether \( R \) is transitive.
5. Determine whether \( R \) is an equivalence relation.

Give a reason why, or a counterexample in each case.